6th Workshop Nonlinear PDEs and Financial Mathematics,
University of Applied Sciences Zittau/Görlitz, Zittau, Germany
23-27 March, 2015

BOOK OF ABSTRACTS

including pages for notes, conference schedule and a plan of Zittau

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SCHEDULE

Day 1. Arrival/Analytics. 23.03.2015,
Building ZIV, Room 0.02, Theodor-Krner-Allee 8, 02763 Zittau

9:00	Steam-train trip to Oybin castle. 2 hour hike. For the participants arriving on the 22^{nd} of March.
11:30	Registration.
12:30	Lunch (Mensa, Hochwaldstr. 12, 02763 Zittau).
13:30	Opening of the conference.
13:30	L. A. Bordag. Optimization problem for a portfolio with an illiquid asset: Lie group analysis
14:15	M. V. Babich. Lie algebras of the complex classical groups and birational Darboux coordinates on their coadjoint orbits.
15:00	I. Kossaczky. Hamilton-Jacobi-Bellman equation
15:30	Coffee.
16:00	D. Puzyrev. Delay-differential equations and large delay approximation
16:30	T. A. Vasilyeva. Valuation of Asia options by implicit difference method
16:50	J. E. Podschipkova. Model of optimal flood hydrograph social natural management of system "Volga Hydroelectric Power Station - Volga-Akhtuba floodplain".
17:10	N. Antonian. Mechanisms of hydrological risk control at the Volga-Akhtuba floodplain
17:30	End of the first day.

Day 2. Stochastic. 24.03.2015,
Building ZII, Room 209, Schliebenstrae 21, 02763 Zittau

9:30	M. do R. Grossinho. Existence results for Nonlinear PDEs arising in Financial Modelling.
10:00	O. Kudryavtsev. Efficient pricing options under Levy processes: a numerical Wiener-Hopf factorization method.
10:30	J. Lueddeckens. Parameter estimation for an electricity price model.
11:00	Coffee.
11:30	Y. Belopolskaya. Probabilistic representations for classical and generalized solutions of the Cauchy problem for quasilinear and fully nonlinear systems of parabolic equations.
12:10	M. F. Agoitia. Modelling electricity prices with polynomial processes.
12:30	Lunch (Mensa, Hochwaldstr. 12, 02763 Zittau).
13:30	A. N. Shiryaev. Bayesian disorder problems on the filtered probability spaces.
14:30	D. B. Rokhlin. Verification problem for viscosity solutions and stochastic Perron's method.
15:30	Coffee.
16:00	Y. Läßig. Estimating seasonalities in energy markets.
16:20	J. Wergieluk. Structural models of spot electricity markets.
16:40	S. Bachmann. Theorem of Yamada, Watanabe and Coupling by reflection for stochastic delay differential equations.
17:00	End of the second day.

Day 3. Numerical methods. 25.03.2015,

Building ZII, Room 209, Schliebenstrae 21, 02763 Zittau

9:30	V. N. Egorova. New fixing-domain transformations for non-linear option pricing models.
10:00	Z. Bučková. ADE Methods - Numerical analysis and application to linear and nonlinear Black-Scholes models.
10:30	G.I. Belyavsky, N. V. Danilova. The fair price calculation of the European option in the case on (B,S)-market model based on the random walk with missing elements.
11:00	Coffee.
11:30	T. A. Vasilyeva. L - stability of thrice implicit difference schemes of eighth order of approximation for ODEs stiff systems.
12:00	M. Ehrhardt. Modeling stochastic correlation.
12:30	Lunch (Mensa, Hochwaldstr. 12, 02763 Zittau).
13:30	I. P. Gavrilyuk. Efficient algorithms for abstract differential equations with applications to PDEs.
14:30	L. G. Vulkov. Compact difference schemes for option pricing liquidity shocks models.
15:00	L. Trussardi. A kinetic equation for modelling irrationality and herding effects.
15:30	Coffee.
16:00	T. Gyulov. On a nonlocal problem for a parabolic integro-differential equation in option pricing with switching liquidity.
16:30	E. Nemchenko. Numerical probabilistic approaches to solution of the Cauchy problem for semilinear parabolic equations.
16:50	A. Grigorieva. A numerical algorithm to construct a generalized solution of a quasilinear system of parabolic equations.
17:10	End of the third day.
17:00	SB meeting of STRIKE (Building ZIII, Room 413).
19:00	Reception of lord mayor of Zittau and rector of the University of Applied Sciences Zittau/Goerlitz in the town hall Zittau (Markt 1, 02763 Zittau).

Day 4. Optimization. 26.03.2015,

Building ZII, Room 209, Schliebenstrae 21, 02763 Zittau

9:30	A. Ellanskaya. Utility maximization and utility indifference price for exponential semi-martingale models and HARA utilities.
10:00	R. Serrano. Martingale approach to utility maximization in jump models with differential rates and marked point processes.
10:30	I. P. Yamshchikov. Portfolio optimization with an exogenous random liquidation time.
11:00	Coffee.
11:30	M. Keller-Ressel. Implied volatilities from strict local martingales.
12:30	Lunch (Mensa, Hochwaldstr. 12, 02763 Zittau).
13:30	J. Merker. On optimal control of systems of elliptic partial differential inequalities.
14:00	S. Pickenhain. Infinite horizon optimal control problems and pseudospectral methods for the solution.
14:30	N. Tauchnitz. Pontryagins maximum principle for infinite horizon optimal control problems with state constraints.
15:00	G. Mironenko. A diffusion stochastic control problem with resource constraints.
15:30	Coffee.
16:00	I. Höfers. Portfolio optimization under dynamic risk constraint.
16:20	M. A. Kharitonov. The operating core of an organization: a constrained optimization model.
16:40	A. Krakhotkin. On minimizing the expected time of beating a benchmark in a factor diffusion model.
17:00	Posters' session. A. Shardin. Partly observable stochastic optimal control problems for an energy storage.
17:20	End of the fourth day.

Day 5. Industry day. 27.03.2015,

Building ZIV, Room 0.01, Theodor-Krner-Allee 8, 02763 Zittau

9:30	A. Tihonov (Head of Search Analytics in Yandex). Cluster analysis. Ticks and tips.
11:00	Coffee.
11:30	Oberbichler (Analyst, Sachsen Asset Management). A practical guide to the analysis of derivatives.
12:30	Lunch (Mensa, Hochwaldstr. 12, 02763 Zittau).
13:30	E. Vasin (Quant at Quantstellation, London) Risk modelling in equities: theory vs practice.
14:30	M. Levin, (Chief Data Scientist - Yandex Data Factory) Advanced Machine Learning in Business
15:30	Coffee. End of the fifth day

Modelling Electricity Prices With Polynomial Processes

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Abstract

We examine particular features of energy spot prices, the so-called stylized facts, such as seasonal patterns, very sharp price spikes, mean reversion and price dependent volatility. Several models have been proposed in the existing literature in order to capture these attributes and we review those which share a special characteristic: they utilize polynomial processes for representing the random (risk) factors.

Polynomial processes were introduced by Cuchiero, Keller-Ressel and Teichmann (2008) and we are going to recall some of their basic and most useful properties.

We will see that some of the most representative models for electricity price dynamics fit into this class of stochastic processes.

Finally, our aim is to describe a more general model obtained through polynomial processes that outperforms the previous ones and yet retains analytical tractability due to the properties of the processes.

The talk is presented on 24th of March

Mechanisms Of Hydrological Risk Control At The Volga-Akhtuba Floodplain.

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Abstract

Environmental safety every year becomes more and more important for society, because there is a correlation between a life quality and environmental safety. Level of environmental safety depends on the choice of society and from a variety of causes that are measurable. Therefore, the environmental risk can be defined as a risk assessment of environmental change for human health, economic activity, political situation and other aspects of society. Under hydrological risk at the Volga-Akhtuba floodplain is defined as the risk of prohibited flooding at the Volga-Akhtuba floodplain and drought in the territories of mandatory flood (environmental risk), the risk of flooding community (social risk) and the risk of material costs (economic risk).

This work describes the mechanisms of hydrological risk control at the Volga-Akhtuba floodplain on various grounds: types and kinds of risks, sources, potential victims, heads of damages, types of risks over time and territorial risks. We create a structural model allows estimation of the aggregated hydrological damages and risks for each of the grounds. For the realization the structural model and the corresponding model of complex assessment are built in the form of hierarchical graph of evaluating. Models of algorithms calculation complex expert estimation, local, complex and hydrological risks of Volga-Akhtuba floodplain are based on the theory of binary bundle. To determine the territorially-localized damages the territory of Volga-Akhtuba floodplain is divided into zones. Local damage in each of the micro zone is considered constant. Also, in each zone is determined by the weight of the local hydrological damage. The estimates of the local hydrological damages are based on data about the flooding of the territory of Volga-Akhtuba floodplain. The data are publicly available. Next are the steps of the method of the complex estimation, which assesses local hydrological damage.

The talk is presented on 23th of March

Lie Algebras Of The Complex Classical Groups And Birational Darboux Coordinates On Their Coadjoint Orbits.

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Abstract

Fundamental "bricks forming the univers" of the Lie groups are *the simple Lie groups.* We consider the classical series of the simple complex Lie groups. The Lie algebras of these groups are the Poisson spaces and the coadjoint action stratify the algebras on the symplectic fibers. Each fiber consists of the elements similar each other, they have the same Jordan structure.

We treat an element of the Lie algebra as a linear transformation of the complex linear space. A set of all linear transformations with a fixed Jordan structure J is a symplectic manifold isomorphic to the coadjoint orbit $\mathcal{O}(J)$ of $\mathrm{SL}(N, \mathbb{C})$.

Any linear transformation can *be projected* parallel to its eigenspace on a coordinate subspace or can *be contracted* on its co-eigenspace. The Jordan structure $\tilde{J}$ of the resulting transformation is defined by the Jordan structure J of the initial transformation, this construction defines *the projection* $\mathcal{O}(J) \to \mathcal{O}(\tilde{J})$ *in question.* I will demonstrate that the fiber $\mathcal{E}$ of the projection is a linear symplectic space and the map $\mathcal{O}(J) \xrightarrow{\sim} \mathcal{E} \times \mathcal{O}(\tilde{J})$ is a birational symplectomorphysm.

The iteration of the procedure gives the isomorphism between $\mathcal{O}(J)$ and the linear symplectic space, which is the direct product of all the fibers of the projections. The Darboux coordinates on $\mathcal{O}(J)$ are pull-backs of the canonical coordinates on the linear spaces in question.

A modification of the method to the groups $\mathrm{SO}(N, \mathbb{C})$ and $\mathrm{Sp}(N, \mathbb{C})$ is presented. One flight of the iteration consists of two actions, namely the projection parallel to a subspace of an eigenspace and the simultaneous contraction on a subspace containing a co-eigenspace. It is found how to choose the couples "eigenspace&co-eigenspace" in such a way that the symmetry of the type $\mathrm{so}(N, \mathbb{C})$ and $\mathrm{sp}(N, \mathbb{C})$ would remain all over the process.

The iteration gives such a set of couples of functions p_k, q_k on the orbit, that the symplectic form of the orbit is equal to $\sum_k dp_k \wedge dq_k$. No restrictions on the Jordan form of the matrices forming the orbit are prescribed.

A coordinate set of functions has been distinguished in the important case of the absence of the non-trivial Jordan blocks corresponding to the zero eigenvalue, it is the case $\dim\ker J = \dim\ker J^2$. The case contains the case of general position, the general diagonalizable case and more.

The talk is presented on 23th of March

Theorem of Yamada, Watanabe and Coupling by Reflection for Stochastic Delay Differential Equations.

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Abstract

In this talk we will consider stochastic delay differential equations. There are two major components to this discussion. Firstly, we analyze the existence and uniqueness of solutions in different senses. For that, we will show that the classical theorem of Yamada and Watanabe is also applicable in the delay case. This allows us to prove the existence of strong solutions on a wider class of stochastic delay differential equations. Furthermore, we obtain strong Markov property for basic assumptions. Secondly, we will focus on the Wasserstein distance between solutions with different initial values. One might expect that these solutions drift apart exponentially in the sense of the Wasserstein metric induced by the usual supremum norm. However, using the technique of coupling by reflection, it will be shown that they approach even exponentially if the norm is modified by a concave function. In addition, we will give a simple example for equations which satisfy the technical assumptions.

The talk is presented on 24th of March

Probabilistic Representations For Classical And Generalized Solutions Of The Cauchy Problem For Quasilinear And Fully Nonlinear Systems Of Parabolic Equations (With Applications To Financial Mathematics).

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Abstract

Nonlinear parabolic equations and systems arize in various problems of financial mathematics as soon as one is going to investigate rather complicate financial markets. In particular, if one wishes to take into account transaction costs and/or the presence on the market of large investors whose investing policy may effect the prices of underlying assets he will immediately come to the necessity to deal with strongly nonlinear parabolic equations to describe the corresponding fair prices of the options (see the Leland model, the Frey-Stremme model and a number of others [1]). In general a fair price $u(t,s)$ of an option with a contract function $\Phi(s)$ and a maturity time T may be found as a solution to the Cauchy problem for a fully nonlinear parabolic equation

$$u_t + g(s,u,u_s,u_{ss}) = 0, \quad u(T,s) = \Phi(s). \tag{1}$$

At the other hand since modern markets have a large number of small investors one has to take into account the so called herd behavior of the investors. To model information herding in a macroscopic setting on stock markets one has to take into account both greed in frenzied buying (bubbles) and fear in selling (crashed). The correspondent model as suggested by Jüngel [2] can be presented for example as a system of equations describing the number $u(t,x)$ of people having information x at time t and the influence potential $v(t,x)$ which have the form

$$\begin{aligned} u_t &= \frac{1}{2}div[\nabla u - \chi(u)\nabla v), \\ v_t &= \frac{1}{2}div(\delta\nabla u + \kappa\nabla v) + vc(u), \\ u(0,x) &= u_0(x), v(0,x) = v_0(x). \end{aligned} \tag{2}$$

The aim of this talk is to construct probabilistic representations of solutions to the Cauchy problems (1) and (2). Let $w(t)$ be a Wiener process defined on a probability space $(\Omega,\mathcal{F},P)$. To give flavor of the presented results we remind that if one considers a nonlinear parabolic equation of the form

$$u_t + \frac{1}{2}A^2(s,u)u_{ss} + a(s,u)u_s + c(s,u)u = 0, \quad u(T,s) = \Phi(s) \tag{3}$$

then he can reduce (3) to a stochastic problem

$$d\xi(\theta) = a(\xi(\theta), u(\theta, \xi(\theta)))d\theta + A(\xi(\theta), u(\theta, \xi(\theta))dw(\theta, \quad \xi(t) = s, \tag{4}$$

$$d\eta(\theta) = c(\xi(\theta), u(\theta, \xi(\theta)))\eta(\theta)d\theta, \quad \eta(t) = 1. \tag{5}$$

$$u(t, s) = E_{t,s,1}[\eta(T)u_0(\xi(T))]. \tag{6}$$

Under suitable assumptions on a, A, c and u_0 one can prove that $u(t, s)$ given by (6) is a unique classical solution of (3). This approach can be extended to deal with some systems of quasilinear and fully nonlinear parabolic equations.

To construct a probabilistic representation to a generalized solution of (2) with constant κ,δ one can deduce from (2) that he needs to deal with a couple of stochastic processes $\hat{\xi}_i(\theta), i = 1, 2$ which are time reversal to solutions $\xi_i(\theta)$ of the following stochastic equations

$$d\xi_1 = \sqrt{1 - \frac{\chi(u(\theta, \xi_1(\theta)))}{u(\theta, \xi_1(\theta)))}})dw(\theta), \tag{7}$$

$$d\xi_2(\theta) = \sqrt{\kappa + \frac{u(\theta, \xi_2(\theta))}{v(\theta, \xi_2(\theta)))}}dw(\theta). \tag{8}$$

To get a closed system one can add relations

$$u_1(t, x) = E_{t,x}[u_{01}(\hat{\xi}_1(t))], u_2(t, x) = E_{t,x}[\eta(T)u_{02}(\hat{\xi}_2(T))]. \tag{9}$$

Under suitable assumptions on $\chi(u), c(u)$ and $u_{0i}(x)$ one can check that there exists a solution to the system (7)–(9) and the functions $u_i(t, x), i = 1, 2$ satisfy (2) in the distributional sense [3],[4].

Note that probabilistic representation of the solutions to systems of nonlinear parabolic equations presented above allow to construct rather effective numerical schemes to obtain numerical solutions of the considered problems.

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References

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The talk is presented on 24th of March

The Fair Price Calculation Of The European Option In The Case Of (B,S)-Market Model Based On The Random Walk With Missing Elements

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Abstract

Let's consider the model class, which generalize the Cox-Ross-Rubinstein model[1]:

$$S_k = S_{k-1} \exp(\delta_k \varepsilon_k + \mu_k) \tag{1}$$

on the standard stochastic basis $F_0 = \sigma(\emptyset, \Omega), F_n = \sigma(S_1, ..., S_n)$, where μ_k - the sequence of predictable stochastic values, ε_k and δ_k - the sequences of independent stochastic values, δ_k - the sequence of binary stochastic values with conditional distribution $p_k = P(\delta_k = 1/F_{k-1})$, where $p_k \in F_{k-1}, \varepsilon_k$ - the sequence of independent and identically distributed stochastic values with distribution law $\nu(dx)$. The formula (1) may be presented as: $S_k = S_0 \exp(\sum_{i=1}^{k} \delta_i \varepsilon_i + \sum_{i=1}^{k} \mu_i), k = 1, .., T$. The sum $\sum_{i=1}^{k} \delta_i \varepsilon_i$ is the random walk with missing elements. The process (1) will be a martingale if:

$$p_k = \frac{\exp(-\mu_k) - 1}{E \exp(\varepsilon_1) - 1}, k = 1, ..., T. \tag{2}$$

From (2) the restrictions on parameters may be obtained:

$$Ee^{\varepsilon_1} \neq 1, \min(1, Ee^{\varepsilon_1}) < e^{-\mu_k} < \max(1, Ee^{\varepsilon_1}) \tag{3}$$

If $\mu_k = \mu$, then probabilities p_k are constants and ν_k are binomially distributed. The significant part in the fair price calculation plays the following sequence of functions:

$$V(k, x) = E_x(f(x \exp(Z_{T-k}))) = E(f(S_T)/S_k = x) \tag{4}$$

where $Z_{T-k} = \sum_{i=1}^{\nu_T - \nu_k} \varepsilon_i + \sum_{i=k+1}^{T} \mu_i$; the conditional expectation is calculated using martingale measure; function f - nonnegative, with definition interval $[0, \infty)$ and finite expectation $Ef(S_T) < \infty$, . With telescopic property of conditional expectation, the following recurrence equations will be obtained:

$$V(T, x) = f(x),$$

$$V(k-1, x) = E_x(k, x \exp(\mu_k(x)) \exp(\delta_k \varepsilon_k)) =$$

$$V(k, x\exp(\mu_k(x)))(1-p_k) + p_k \int_{-\infty}^{+\infty} V(k, x\exp(\mu_k(x))\exp(y))\nu(dy) \tag{5}$$

When $\mu_k(x) = \mu$ the recurrence formulas may be simplified:

$$V(k,x) = \sum_{j=0}^{T-k} C_{T-k}^{j} p^j (1-p)^{T-k-j} \int_{-\infty}^{+\infty} f(x\exp(y + (T-k)\mu))\nu^j(dy) \tag{6}$$

where $\nu^j(dy)$ - the compression of distributions, $\nu_0(dy) = \delta_0(dy), \delta_0$ - the Dirak measure. Let‘s apply the Fourier integral for the formula (6).

Let‘s identify $\varphi(x,y) = f(xexp(y)), x > 0$ and assume that there exists such positive value ρ that function $\psi(x,y) = \exp(-\rho y)\varphi(x,y)$ has direct and inverse Fourier transform with fixed x and $E\exp(\rho\varepsilon_1) < \infty$. If $\overline{\psi}$ - the Fourier transform of function ψ, then

$$V(k,x) = \int_{-\infty}^{+\infty} \overline{\psi}(x,y)\exp(i(T-k)\mu(y-i\rho))(1-p+p\eta_\varepsilon(y-i\rho))^{T-k} dy \tag{7}$$

Let‘s apply the formula (7) for the European call option. For this option the function $f(x) = \max(0, x-K)$ where K - the contract price, function $\phi(x,y) = \max(0, x\exp(y) - K)$. The Fourier transform of function ψ is the following:

$$\overline{\psi}(x,y) = \frac{K(K/x)^{-\rho-iy}}{2\pi(\rho+iy-1)(\rho+iy)} \tag{8}$$

So,

$$V(k,x) = \frac{K}{2\pi i}\int_{-\infty-i\rho}^{+\infty-i\rho} \frac{\exp(iy((T-k)\mu - ln(K/x)))}{y(iy-1)}(1-p+p\eta_\varepsilon(y))^{T-k} dy \tag{9}$$

The formula (9) is the generalized Cox-Ross-Rubinstein formula.

Example 1. Let‘s consider the symmetric random walk with missing elements and constant μ. In this example the characteristic function is the following: $\eta_\varepsilon(y) = \cos(\alpha y)$, the martingale probability is the following: $p = P(\delta_k = 1) = \dfrac{2(\exp(-\mu)-1)}{\exp(\alpha)+\exp(-\alpha)-1}$. The formula (6) may be presented in the following form:

$$V(k,x) = \sum_{i=0}^{T-k} C_{T-k}^{i}\left(\frac{1}{2^i}\sum_{m=0}^{i} C_i^m \max(0, x\exp((T-k)\mu + \alpha(2m-i)) - K)\right) \times$$

$$p^i(1-p)^{T-k-i} \tag{10}$$

Let $T = 10, \rho = 1.5, S_0 = K = 32, \alpha\dfrac{365}{ln(1.1)} = 10, -\mu\dfrac{365}{ln(1.00001)} = 10$. Then $V(0,32) = 0.025067$(the Fourier integral), $V(0,32) = 0.024584$(the exact formula).

The recurrence formulas (5) evaluations may be made with tree method [2]. The tree method uses the approximate model. Let‘s define the sets $H = (j\alpha, j = \pm1, \pm2, ...), G = (\exp(j\alpha), j = \pm1, \pm2, ...)$ and the sequence of independent and identically distributed stochastic values $(\overline{\varepsilon}_i)_{i=1}^N$ such as $\overline{\varepsilon}_i = \alpha\sum_j jI(j\alpha \le \varepsilon_i \le (j+1)\alpha)$, which approximates the sequence $(\varepsilon_i)_{i=1}^N$ such as $P(\overline{\varepsilon} = j\alpha) = \displaystyle\int_{j\alpha}^{(j+1)\alpha}\nu(dx)$. Let $\overline{\mu}_k(\overline{S}_{k-1}) = \alpha\sum_j jI((j-1)\alpha \le$

$\mu_k(\overline{S}_{k-1}) \le j\alpha)$ and $S_0 \in G$. In this case the process $S_k = S_{k-1}\exp(\delta_k\overline{\varepsilon}_k + \overline{\mu}_k(\overline{S}_{k-1})), k = 1, ..., T$ approximates the Markov chain S with phase space G. The function $\overline{V}(k,x)$ approximates the function $V(k,x)$, which satisfies the equation $E_x(f(x\exp(\overline{Z}_{T-k}))) = E(f(\overline{S}_T)/\overline{S}_k = x)$. Let's identify the function $\overline{V}$ in the lattice point: $\overline{V}_{k,j\alpha} = \overline{V}(k, \exp(j\alpha))$. The recurrent equations (5) for the approximate sequence will be the following:

$$V_{T,j\alpha} = f(\exp(j\alpha)),$$

$$V_{k-1,j\alpha} = V_{k,j\alpha+\overline{\mu}_k(\exp(j\alpha))}(1-p_k) + p_k\left(\sum_i V_{k,j\alpha+\overline{\mu}_k(\exp(j\alpha))} + i\alpha\int_{(i-1)\alpha}^{i\alpha}\nu(dy)\right) \tag{11}$$

Example 2. Let's consider the symmetric random walk with missing elements and $\mu_k(S_{k-1}) = \left(\dfrac{-3h(e^{\alpha}+e^{-\alpha})}{8}\right) I(S_{k-1} \ge \theta_1) + \left(\dfrac{-h(e^{\alpha}+e^{-\alpha})}{4}\right) I(\theta_2 < S_{k-1} < \theta_1) + \left(\dfrac{-h(e^{\alpha}+e^{-\alpha})}{4}\right) I(S_{k-1} \le \theta_2)$. For this example the formulas (11) will be the following:

$$V_{T,j\alpha} = f(\exp(j\alpha))$$

$$V_{k-1,j\alpha} = V_{k,j\alpha+\overline{\mu}_k(\exp(j\alpha))}(1-p_k) + (p_k/2)(V_{k,j\alpha+\overline{\mu}_k(\exp(j\alpha))-\alpha} + V_{k,j\alpha+\overline{\mu}_k(\exp(j\alpha))+\alpha}) \tag{12}$$

Let $S_0 = K = 32, \alpha = 0.03, h = 0.1, T = 10, \theta_1 = 32.5, \theta_2 = 31.5$. Then $V(0,32) = 0.023369$. If the corridor is wide, then the price is large.

References

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The talk is presented on 25th of March

Optimization Problem For A Portfolio With An Illiquid Asset: Lie Group Analysis.

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Abstract

Management of a portfolio that includes an illiquid asset is an important problem of modern mathematical finance. One of the ways to model illiquidity among others is to build an optimization problem and assume that one of the assets in a portfolio cannot be sold until a certain finite, infinite or random moment of time. This framework usually leads to a three-dimensional Hamilton-Jacobi-Bellman (HJB) equation on the value function. These nonlinear PDEs are then studied with different analytical and numeric methods.

This approach arises a certain amount of models that are actively studied at the moment. These models use different utility functions or liquidation time distributions for an illiquid asset. To reduce the three-dimensional problem to a two-dimensional one or even to an ODE one uses some substitutions. The types of possible substitutions are used commonly yet were never profoundly studied before to our knowledge.

We carry out a complete Lie group analysis of PDEs that arises for four different portfolio optimization problems. We take two different utility functions: HARA- and Log- utility function as well as two different exogenous liquidation time distributions : an exponential and a Weibull distribution.

We find the admitted Lie algebras for HJB equations describing value function and investment and consumption strategies for a portfolio with illiquid asset that is sold in a random moment of time with a prescribed distribution.

We describe the optimal system of subalgebras for all these four cases, which give rise to non-trivial, non-equivalent substitutions in HJB equation. We describe all non-equivalent substitutions; provide the reductions and the corresponding lower dimensional equations. These lower dimensional equations can be used for further studies of portfolio optimization problems.

Several of these substitutions were used in other papers before for the exponential liquidation time distribution and other ones are new. This method gives us the possibility to provide a complete set of non-equivalent substitutions and reduced equations.

The talk is presented on 23th of March

ADE Methods - Numerical Analysis And Application To Linear And Nonlinear Black-Scholes Models.

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Abstract

We are dealing with numerical methods for linear and nonlinear Black- Scholes model. We apply finite difference method, esp. Alternating direction explicit methods (ADE), which were suggested in 1957 by Saul'ev. Our work includes detailed numerical analysis consisting of stability and consistency proofs.

Numerical results of the ADE method for nonlinear Black-Scholes models, where the nonlinearity is caused by illiquid markets, such as Frey and Patie model, are provided.

We compare our method to alternative numerical approaches for solving the nonlinear Black-Scholes equation from the literature.

The talk is presented on 25th of March

New Fixing-Domain Transformations For Non-Linear Option Pricing Models.

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Abstract

Non-linear American option pricing models (see [2], [3]) are considered. In order to solve the free-boundary partial differential equation a front-fixing method [1] is proposed. Because of the non-linearity for the numerical solution can be used an implicit finite difference method. In order to construct explicit finite difference scheme a new moving domain transformation is proposed. Numerical algorithms are developed for solve both the option price and the optimal exercise boundary.

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The talk is presented on 25th of March

Modelling Stochastic Correlation.

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Abstract

This work deals with the modelling of correlation. It is well known that the correlation between financial products, financial institutions, e.g., plays an essential role in pricing and evaluation of financial derivatives. Using simply a constant or deterministic correlation may lead to correlation risk, since market observations give evidence that the correlation is not a deterministic quantity.

In this work, we describe a new approach to model the correlation as a hyperbolic function of a stochastic process. Our general approach provides a stochastic correlation which is much more realistic to model real world phenomena and could be used in many financial application fields. Furthermore, it is very flexible: any mean reverting process (with positive and negative value) can be regarded and no additional parameter restrictions appear which simplifies the calibration procedure.

As an example, we compute the price of a quanto applying our new approach.

Using our numerical results we discuss concisely the effect of considering stochastic correlation on pricing the quanto.

The talk is presented on 25th of March

Utility Maximization And Utility Indifference Price For Exponential Semi-martingale Models And HARA Utilities.

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Abstract

We consider the utility maximization problem for semi-martingale models and HARA (hyperbolic absolute risk aversion) utilities. Using specific properties of HARA utilities, we reduce the initial maximization problem to the conditional one, which we solve by applying a dual approach. Then we express the solution of the conditional maximization problem via conditional information quantities related to HARA utilities, like the Kullback-Leibler information and Hellinger-type integrals. In turn, we express the information quantities in terms of information processes, which is helpful in indifference price calculus. Finally, we give equations for indifference prices. We apply the results to Black-Scholes model with correlated Brownian motions, jump-diffusion model and Lévy model and give an explicit expression for information quantities. Then the previous formulas for the indifference price can be applied.

The talk is presented on 26th of March

Efficient Algorithms For Abstract Differential Equations With Applications To PDEs.

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Abstract

We discuss some novel numerical algorithms for linear and nonlinear differential equations with operator coefficients in a Banach space. These equations can be viewed as meta-models for PDE's. As examples of such equations, including the parabolic PDEs, one can consider the following first order differential equations

$$u'(t) + Au = f(t), u(t_0) = u_0 \tag{1}$$

or

$$u'(t) + Au = f(t, u), u(t_0) = u_0, \tag{2}$$

where u_0 is a given element of a Banach space X, $u(t) : \mathbb{R}_+ = [0, \infty) \to X$ is the unknown function with values in X, $f(t), f(t, u)$ are given functions $\mathbb{R}_+ = [0, \infty) \to X, \mathbb{R}_+ \times \mathbb{R}_+ \to X$ respectively and $A : D(A) \subset X \to X$ is a given linear operator. As a special example of (1) can serve the well known heat equation

$$\begin{aligned} &\frac{\partial u(t,x)}{\partial t} - \frac{\partial^2 u(t,x)}{\partial x^2} = f(t,x),\ t > 0, x \in (0,1) \\ &u(t,0) = 0,\ u(t,1) = 0, \\ &u(0,x) = u_0(x). \end{aligned} \tag{3}$$

Here the operator A is defined by

$$\begin{aligned} &D(A) = \{v(x) : v(x) \in H^2(0,1), v(0) = v(1) = 0\}, \\ &Av = -v''(x),\ \forall\, v \in D(A). \end{aligned} \tag{4}$$

The well known Black-Scholes-Merton model, which represents a mathematical model of a financial market containing certain derivative investment instruments and describes the price of the option over time, can be transformed to the heat equation.

In the linear case the solution of abstract differential equations of various order can be represented through the solution operator $S = S(t) = S(t, A)$ by $u(t) = S(t)u_0$. In the case of the first order differential equation (1) the solution operator is the semigroup generated by A or the so called operator exponential $S(t) = e^{-At}$.

The often used approximation methods are the finite difference (FD) and the finite element (FEM) methods, which possess a polynomial convergence rate with respect to the number N of unknown parameters (the approximate values of the solution at the grid points) and can not provide the optimal accuracy predicted by the N-width theory and, therefore, the optimal complexity.

Our algorithms are based on the ideas of Sinc quadratures, of homotopy and on the Cayley transform. They possess an exponential convergence (accuracy) rate, optimal or

near optimal complexity and can be parallelized. A proper approximation of the solution operators, which are functions of the operator coefficients of the abstract differential equations, plays a crucial role.

Examples are presented to support the theory.

The talk is presented on 25th of March

A Numerical Algorithm To Construct A Generalized Solution Of A Quasilinear System Of Parabolic Equations.

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Abstract

Systems of quasilinear parabolic equations arise as mathematical models of various financial and biological processes such as growth of cell populations under contact inhibition or as models describing density evolution dynamics for interacting populations of prey-predator type as well as various socio-economical processes such as price equalizing process or the processes describing herd behavior of investors [1], or as price dynamics process at the markets with trading costs and illiquidity [2].

The aim of this talk is to discuss numerical algorithms of constructing generalized solutions of the Cauchy problem for systems of fully coupled nonlinear parabolic equations of the form

$$u^i_t = (u^i(u^1+u^2))_{xx} + u^i(a_i - b_i u^1 - c_i u^2), \quad u^i(0,x) = u^i_0(x), \quad i = 1,2. \tag{1}$$

We say that functions u^1, u^2 denote generalized solutions of the system (1) if they belong to the Sobolev class $H^{1,2}_2([0,T]\times R)$ and satisfy integral identities

$$\langle h(t), u^i(t)\rangle - \langle h(0), u^i(0)\rangle = \int_0^t \langle (a_i - b_i u^1(s) - c_i u^2(s))h(s), u^i(s)\rangle ds \tag{2}$$

$$+ \int_0^t \langle [\partial_s h(s) + \frac{1}{2}(u^1(s)+u^2(s))h_{xx}(s)], u^i(s)\rangle ds, \quad i = 1,2.$$

where $\langle u, h\rangle = \int_R u(x)h(x)dx$.

As it was shown in [3] to obtain a probabilistic representation of a regular generalized solution u^1, u^2 to (1), we may consider a stochastic equation

$$d\xi(\theta) = -M^{u^1,u^2}(\theta, \xi(\theta))dw(\theta), \quad \xi(0) = y, \tag{3}$$

where $M^{u^1,u^2}(\theta,\xi(\theta)) = \sqrt{u^1(\theta,\xi(\theta)) + u^2(\theta,\xi(\theta))}$ and $w(t) \in R^1$ is a standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, P)$. Then setting $g^i(u)(t,x) = u^i(t,x)(a_i - b_i u^1(t,x) - c_i u^2(t,x))$ and $\hat\xi(\theta) = \xi(t-\theta)$ with $\hat\xi(0) = x$ we can check that the relation

$$u^i(t,x) = E_{0,x}[\hat\eta(t) u_{0i}(\hat\xi^i(t))], \quad i = 1,2 \tag{4}$$

where the process $\hat\eta(\theta)$ satisfies a linear ODE

$$d\hat\eta^i(\theta) = g_i^{u^1,u^2}(\theta, \hat\xi(\theta))\hat\eta^i(\theta)d\theta, \quad \hat\eta^i(0) = 1 \tag{5}$$

defines the required probabilistic representation.

Similar to [5] we construct a numerical algorithm based on the probabilistic representation developed in [3] of the required solution . To this end we need a stochastic equation that governs the process $\hat{\xi}(t)$. The correspondent equation has the form

$$\hat{\xi}(\theta) = x + \int_0^\theta m^{u^1,u^2}(s,\hat{\xi}(s))ds + \int_0^\theta M^{u^1,u^2}(s,\hat{\xi}(s))d\tilde{w}(s), \tag{6}$$

where $\tilde{w}(s) = w(t-s) - w(t)$ and $m^{u^1,u^2}(s,\hat{\xi}(s)) = \frac{1}{2}[u_x^1(s,\hat{\xi}(s)) + u_x^2(s,\hat{\xi}(s))]$. To make the system closed we need as well two more equations, namely,

$$d\hat{J}(\theta) = b^{u^1,u^2}(\theta,\hat{\xi}(\theta))\hat{J}(\theta)d\theta + B^{u^1,u^2}(\theta,\hat{\xi}(\theta))\hat{J}(\theta)d\tilde{w}(\theta), \quad \hat{J}(0) = I, \tag{7}$$

where

$$B^{u^1,u^2}(\theta,x) = \frac{1}{2}\frac{u_x^1(\theta,x) + u_x^2(\theta,x)}{\sqrt{u^1(\theta,x) + u^2(\theta,x)}}, \quad b^{u^1,u^2}(\theta,x) = \frac{[u_x^1(\theta,\hat{\xi}(\theta)) + u_x^2(\theta,x)]^2}{u^1(\theta,x) + u^2(\theta,x)}$$

and

$$\frac{\partial u^i(t,x)}{\partial x} = \mathbf{E}\left[\int_0^t \frac{\partial g_i^{u^1,u^2}(\hat{\xi}_{0,x}(\theta))}{\partial x}\hat{J}(\theta)d\theta exp\left\{\int_0^t g_i^{u^1,u^2}(\hat{\xi}_{0,x}(\theta))d\theta\right\} u_{0i}(\hat{\xi}_{0,x}(t))\right] + \tag{8}$$

$$\mathbf{E}\left[exp\left\{\int_0^t g_i^{u^1,u^2}(\hat{\xi}_{0,x}(\theta))d\theta\right\}\frac{\partial u_{0i}(\hat{\xi}_{0,x}(t))}{\partial x}\hat{J}(t)\right], \; i = 1,2.$$

We construct a layer method to get a numerical solution of the Cauchy problem for the system (1) and present it here choosing for simplicity $g^i \equiv 0$ and putting $\gamma(t) = \hat{\xi}(t)), \lambda(t) = \hat{J}(t)$.

To this end we consider the equidistant time discretization . $0 \leq t_0 \leq \cdots \leq t = t_n$, $\Delta t = \frac{t}{n}$, and approximate $\Delta_k w(t) = w(t_k + \Delta t) - w(t_k)$ by $\sqrt{\Delta t}\zeta_k$, where $P(\zeta_k = \pm 1) = \frac{1}{2}$. We deduce from (5) (since $\hat{\eta}(t) = 1$)

$$u^i(t_k,x) = E_{t_k,x}[u^i(t_{k+1},\hat{\xi}(t_{k+1}))].$$

Applying the explicit Euler scheme to (6) and (7) we get

$$\gamma(t_{k+1}) \sim \bar{\gamma}(t_{k+1}) = x + m^{\bar{u}^1,\bar{u}^2}(t_k,x)\sqrt{\Delta}t + M^{\bar{u}^1,\bar{u}^2}(t_k,x))\sqrt{\Delta}t\zeta_k$$

where $\zeta_k, k = 0,\ldots n-1$ are i.i.d random variables distributed by the law $P(\zeta_k = \pm 1) = \frac{1}{2}$.

$$\lambda(t_{k+1}) \sim \bar{\lambda}(t_{k+1}) = 1 + b^{\bar{u}^1,\bar{u}^2}(t_k,x)\bar{\lambda}(t_k)\sqrt{\Delta}t + B^{\bar{u}^1,\bar{u}^2}(t_k,x))\bar{\lambda}(t_k)\sqrt{\Delta}t\zeta_k.$$

Using (5) and (8) we obtain

$$u^i(t_k,x) \sim E[\bar{u}^i(t_{k+1},\bar{\gamma}_{t_k,x}(t_{k+1}))] =$$

$$\frac{1}{2}u^i(t_{k+1}, x + m^{u_1,u^2}(\bar{u}^1(t_k,x) + \bar{u}^2(t_k,x)) + M^{\bar{u}^1,\bar{u}^2}(t_k,x))\sqrt{\Delta}t)$$

$$+\frac{1}{2}u^i(t_{k+1}, x + m^{\bar{u}_1,\bar{u}^2}(t_k,x) + M^{\bar{u}^1,\bar{u}^2}(t_k,x))\sqrt{\Delta}t)$$

and a similar approximation can be evaluated for

$$u_x^i(t_k,x) \sim E[\bar{u}_x^i(t_{k+1},\bar{\gamma}_{t_k,x}(t_{k+1}))\bar{\lambda}(t_{k+1})].$$

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The talk is presented on 25th of March

Existence Results For Nonlinear PDEs Arising In Financial Modelling.

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Abstract

Black-Scholes equation has been widely used by academicians and practitioners when studying contingent claims on underlying securities. When considering stochastic volatility or transaction costs we can be lead to fully nonlinear parabolic equations.

We are concerned with the existence and localization of stationary solutions of nonlinear versions of the standard parabolic Black-Scholes. The argument used is based on the upper and lower solutions method.

With this study, we aim to contribute for a better understanding of some analytical features of some problems that arise in financial modelling. We refer that stationary solutions can become interesting in finance when the time does not play a relevant role such as, for instance, in perpetual options.

The talk is presented on 24th of March

On A Nonlocal Problem For A Parabolic Integro-Differential Equation In Option Pricing With Switching Liquidity.

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Abstract

A nonlinear parabolic integro-differential equation

$$\frac{\partial u}{\partial \tau} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 u}{\partial S^2} = -\nu_{01} e^{u(S,\tau)} \left(\nu_{10} \int_0^{\tau} e^{-u(S,s)} ds + e^{-\gamma h(S)} \right) + \kappa,$$

is studied. The problem arises from the option pricing in a financial market with changing liquidity. Here $S \in [0, S_{\max}]$ denotes the price of the underlying asset, $\tau = T - t \in [0, T]$ is the time to maturity T and σ, ν_{01}, ν_{10}, γ and κ are parameters. The unknown function u satisfies the initial condition $u(S,0) = \gamma h(S)$ for a given payoff function $h(S)$ and has a prescribed time-dependent mean value

$$\frac{1}{S_{\max}} \int_0^{S_{\max}} u(S,\tau) dS = \varphi(\tau)$$

where $\varphi(\tau)$ is some given function.

The talk is presented on 25th of March

Portfolio Optimization Under Dynamic Risk Constraints.

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Abstract

We consider an investor faced with the classical portfolio problem of optimal investment in a log-Brownian share and a fixed-interest bond, but constrained to choose portfolio and consumption strategies which reduce the corresponding shortfall risk. Risk measures are calculated for short time intervals and imposed on the strategy as a risk constraint. To derive optimal strategies under this constraint, we apply dynamic programming techniques and combine the resulting Hamilton-Jacobi-Bellman equation with the method of Lagrange multipliers to handle the constraint. An approximate solution to the constrained portfolio problem is obtained by using a policy improvement algorithm. In addition we present various numerical methods to solve the partial differential equations arising in this algorithm. Our numerical results indicate that the effect of the risk constraint is very small, that is the investor is not losing very much compared to the unconstrained case.

The talk is presented on 26th of March

Implied Volatilities From Strict Local Martingales.

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Abstract

Several authors have proposed to model price bubbles in stock markets by specifying a strict local martingale for the risk-neutral stock price process. Such models are consistent with absence of arbitrage (in the NFLVR sense) while allowing fundamental prices to diverge from actual prices and thus modeling investors exuberance during the appearance of a bubble. We show that the strict local martingale property as well as the "distance to a true martingale" can be detected from the asymptotic behavior of implied option volatilities for large strikes, thus providing a model-free asymptotic test for the strict local martingale property of the underlying. This talk is based on joint work with Antoine Jacquier.

The talk is presented on 26th of March

The Operating Core Of An Organization: A Constrained Optimization Model.

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Abstract

This report suggests a constrained optimization model for the operating core of an organization. Structurally, the core consists of the basic technological module and several modules of support facilities. The production function of the operating core is represented through the superposition of Leontief production functions corresponding to each of the modules. We reduce the optimization problem to a linear programming problem with a parameter describing the operational structure of the core. In addition, we develop a certain algorithm for automatic construction of the basic equations for each value of the parameter. Finally, the optimization problem is solved numerically for a wide range of the model variables and parameters. The proposed model may serve for designing generalized control mechanisms for an organizational system on a large horizon.

The talk is presented on 26th of March

Hamilton-Jacobi-Bellman equation.

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Abstract

Hamilton-Jacobi-Bellman equation is nonlinear partial differential equation, arising typically from continuous optimal control problems. This equation can be seen as a continuous equivalent of discrete Bellman principle. In this presentation, we will take a closer look at the role of Hamilton-Jacobi-Bellman (HJB) equation in financial mathematics. Analytical solution of the HJB equation is rarely feasible, therefore numerical methods should be used. In order to cope with the nonlinearity of HJB equation, special conditions should be satisfied.

The talk is presented on 23th of March

On Minimizing The Expected Time Of Beating A Benchmark In A Factor Diffusion Model.

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Abstract

1. The study of optimal investment problems with a benchmark in diffusion models was initiated by Browne [2; 3]. We consider one of the problems posed in [3], where the aim of an investor is to outperform a stochastic target in minimum time. In our model the capital of the investor X is affected by a random factor Y:

$$dX_t = X_t(r + \pi_t(\mu(Y_t) - r))dt + X_t\pi_t\sigma(Y_t)dW_t,$$

$$dY_t = g(Y_t)dt + h(Y_t)dW'_t,$$

where W, W' are independent standard Brownian motions. An investment strategy $\pi_t \in [\underline{\pi}, \overline{\pi}]$ is a progressively measurable process with respect to the P-augmented natural filtration of W, W'.
Consider the ratio $Z = X/L$, where L is a stochastic benchmark satisfying the equation

$$dL_t = \alpha L_t dt + bL_t dW_t + \beta L_t dW''_t$$

with W'' independent from W, W'. By Ito's formula we obtain the following equation for the ratio process:

$$dZ_t = Z_t(\hat{r} + \pi_t\hat{\mu})dt + Z_t(\pi_t\sigma(Y_t) - b)dW_t - Z_t\beta dW''_t,$$

where $\hat{r} = r - \alpha + b^2 + \beta^2$ and $\hat{\mu} = \mu(Y_t) - r - \sigma(Y_t)b$. We assume that μ, σ, g, h satisfy Lipschitz and linear growth conditions.

Denote by τ the first exit time of the process Z from the interval $[l, u]$, where $0 \le l < u < \infty$. We consider the risk-sensitive type criterion (see [4])

$$v(z, y) = \sup_{\pi\in[\underline{\pi},\overline{\pi}]} \mathsf{E}(e^{-\lambda\tau} I_{\{Z_\tau = u\}}), \quad \lambda > 0$$

related to the minimization of $\mathsf{E}(\tau I_{\{Z_\tau = u\}})$.

2. Using the known results from the theory of stochastic optimal control (see, in particular, [1; 6]), we prove that the value function v is the unique bounded viscosity solution of the Hamilton-Jacobi-Bellman equation:

$$\lambda v - g(y)v_y - \tfrac{1}{2}h^2(y)v_{yy} - \hat{r}zv_z - \tfrac{1}{2}(b^2 + \beta^2)z^2 v_{zz}$$

$$- \sup_{\pi\in[\underline{\pi},\overline{\pi}]} \left\{ \pi\hat{\mu}(y)zv_z + \frac{1}{2}(\pi^2\sigma^2(y) - 2\pi\sigma b)z^2 v_{zz} \right\} = 0, \quad (z, y) \in (l, u) \times \mathbb{R},$$

which is continuous on $[l, u] \times \mathbb{R}$ and satisfies boundary conditions $v(l, y) = 0, v(u, y) = 1$. Note, that v attains 0 at the boundary points $z = l$ continuously even in the case $l = 0$, where the diffusion degenerates at the normal direction to this part of the boundary.

To solve the problem numerically we use monotone difference schemes [5]. Their convergence follows from the comparison result (see [6], Theorem 6.21). The numerical experiments where performed for the Ornstein-Uhlenbeck process Y.

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The talk is presented on 26th of March

Efficient Pricing Options Under Levy Processes: A Numerical Wiener-Hopf Factorization Method.

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Abstract

In recent years more and more attention has been given to stochastic models of financial markets which depart from the traditional Black-Scholes model. We concentrate on one-factor non-Gaussian exponential Levy models. These models provide a better fit to empirical asset price distributions that typically have fatter tails than Gaussian ones, admit jumps and can reproduce volatility smile phenomena in option prices.

We propose a general efficient numerical approach for pricing options under Levy processes. In the case of a wide class of options (e.g. barrier, lookback, American or swing options), Carr's randomization or the Laplace transform reduces the pricing problem to the calculation of the appropriate sequence of stationary generalized Black-Scholes equations subject to the correspondent boundary conditions. Each equation can be easily solved using a fast and accurate numerical Wiener-Hopf factorization method (the FWHF-method). The FWHF-method is based on an efficient approximation of the Wiener-Hopf factors in the exact formula for the solution and the Fast Fourier Transform algorithm. In contrast to finite difference methods which require a detailed analysis of the underlying Levy model, the FWHF-method deals with the characteristic exponent of the process. We show the advantage of the method suggested in terms of accuracy and convergence by using finite difference schemes and Monte-Carlo simulations.

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The talk is presented on 24th of March

Estimating Seasonalities In Energy Markets.

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Abstract

Seasonalities are an important component of energy prices, in particular of electricity spot prices. We propose a new approach combining periodic factors with trigonometric functions and a piecewise affine price trend with change points. This includes a number of well-known models for seasonalities. Our main goal is to obtain an efficient estimate of the seasonalities.

In this regard, we study a GARCH model with seasonal components and estimate the coefficients jointly with the seasonalities via a quasi-maximum likelihood approach and, as an alternative, via the EM algorithm. This procedure outperforms a separate estimation of the seasonalities and the GARCH coefficients of the deseasonalized time series.

The talk is presented on 24th of March

Advanced Machine Learning in Business

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Abstract

There's big hype about Big Data for the last few years. There are many intriguing success stories described in the press for mass audience without too many details. Some people and companies are skeptical about all that hype. Is there really any advantage in the Big Data? Is the data really big? What are the industries where it can be and cannot be applied? How to find those applications? Are advanced algorithms and math really needed, or can everything be done good enough using a good toolbox of standard algorithms? Do you have to be a domain expert to apply machine learning successfully? I'll tell you a few cases about practical applications of advanced machine learning to real business cases, which will answer all those questions, at least in part.

The talk is presented on 27th of March

Parameter Estimation For An Electricity Price Model.

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Abstract

The ongoing liberalization of the European electricity market and the growing share of renewable energy in electricity generation induce a higher difficulty in predicting the development of electricity prices. Therefore it is necessary to enhance existing models.

In this talk we introduce a new mathematical model of the electricity price dynamics. Our model meets better the requirements of the electricity market because it involves the fractional character, the seasonal and periodic as well as the short-term and long-term volatility of electricity prices. Mathematically, this can be described by a stochastic differential equation driven by a fractional Brownian motion and a compensated Poisson random measure. In previous studies [1] we have proved the existence and uniqueness of the solution process and have shown an explicit solution of this differential equation. In this talk we develop a method for parameter estimation for this new model and prove its consistency.

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The talk is presented on 24th of March

On Optimal Control Of Systems Of Elliptic Partial Differential Inequalities.

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Abstract

Optimal control of systems governed by elliptic PDEs has many applications and is the topic of many monographs, see e.g. [1-3]. Often the objective is a coercive functional and the constraints are scalar (semi)linear elliptic PDEs, as e.g. in distributed control of elliptic PDEs with quadratic objective. In contrast, in case of a linear objective, a quadratic system of linear elliptic partial differential inequalities and a sign condition on the state variables, the optimal control problem reads as

$$\begin{aligned} &\int_\Omega \sum_{i=0}^{I} g_i u_i dx = \max! \quad \text{under constraits} \\ &-\mathrm{div}(\sum_{j=1}^{I} a_{ij} \nabla u_j) + \sum_{j=1}^{I} b_{ij} u_j \leq f_i \text{ and } u_i \geq 0 \text{ in } \Omega \text{ for every } i = 1, ..., I \end{aligned} \tag{1}$$

under Dirichlet or Neumann conditions on the boundary $\partial\Omega$ of the bounded domain Ω. Such problems arise e.g. in business mathematics as generalizations of linear programs for production planning, if the products and resources are spatially distributed and a lack of products in some subregion $\Omega' \subset \Omega$ leads to a flow of resources over the boundary $\partial\Omega'$, or in chemistry/biology, if activators resp. inhibitors are used in a planned reaction-diffusion process to obtain e.g. a maximal concentration of u_i in a fixed subregion Ω_i (then $g_i := 1_{\Omega_i}$). Infinite-dimensional linear programs [4] like (1) can be written in an abstract way as $\langle g, u\rangle = \max!$ under constraints $A_u \leq f$ and $u \geq 0$ with a linear operator $A : V \to V^*$ on an ordered Hilbert space $(V, \leq)$. In the talk, the existence of optimizers is shown under a rather general condition about the coercivity of A on the non-negative cone, and best coercivity constants are studied. Finally, duality and extensions to quasilinear differential operators A are discussed.

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The talk is presented on 26th of March

A Diffusion Stochastic Control Problem With Resource Constraints.

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Abstract

Consider a controlled diffusion process $X = (X^1, ..., X^n)$, governed by the system of stochastic differential equations

$$dX_t = b(X_t, \alpha_t)dt + \sigma(X_t, \alpha_t)dW_t, \quad X_0 = x,$$

where $W = (W^1, ..., W^m)$ is a standard Brownian motion with respect to some filtration $\mathbb{F} = (\mathscr{F}_s)_{s \geq 0}$, satisfying the usual conditions. We assume that the problem is in the standard form [5, Chapter 3]. An $\mathbb{F}$-progressively measurable process $\alpha \in [\underline{u}, \overline{u}]$, $\underline{u} < 0 < \overline{u}$ is considered as an intensity of resource consumption. The resource amount Y satisfies the equation

$$dY_t = -|\alpha_t|dt, \quad Y_0 = y.$$

We call a process α *admissible*, and write $\alpha \in \mathcal{A}(x, y)$, if $Y_t \geq 0, \ t \geq 0$. The resource Y may be considered as a fuel amount or as a currency reserve. In the later case α is the intensity of currency interventions.

Let $\theta^{x,y,\alpha}$ be the exit time of X from a bounded set $G \subset \mathbb{R}^n$:

$$\theta^{x,y,\alpha} = \inf\{t \geq 0 : X_t^{x,y,\alpha} \notin G\}.$$

The objective functional J and the value function v are defined by

$$J(x, y, \alpha) = \mathsf{E} \int_0^{\theta^{x,y,\alpha}} e^{-\beta t} f(X_t^{x,y,\alpha}, \alpha_t)dt, \quad v(x, y) = \sup_{\alpha \in \mathcal{A}(x,y)} J(x, y, \alpha),$$

where $\beta > 0$, and f is a bounded continuous function.

Note, that by the definition of an admissible strategy, one have to put $\alpha = 0$, as far as "he fuel is exhausted", that is, as Y reaches the level 0. In particular, if $Y_0 = 0$ then the value function is determined by the correspondent uncontrolled problem with $\alpha = 0$. In this case v coincides this the solution of the Dirichlet problem for the linear elliptic differential equation:

$$\beta\psi - f(x, 0) - b(x, 0)\psi_x - \frac{1}{2}\mathrm{Tr}(\sigma(x, 0)\sigma^T(x, 0)\psi_{xx}) = 0, \quad x \in G; \quad \psi = 0 \text{ on } \partial G,$$

by the well-known probabilistic representation of its solution (under the assumption, that the matrix $\sigma(x, 0)\sigma^T(x, 0)$ is non-degenerate).

By applying these considerations, it is easy to show that one can pass to the equivalent *exit time* control problem:

$$v(x,y) = \sup_{\alpha \in \mathcal{U}} \mathsf{E} \left(\int_0^{T^{x,y,\alpha}} e^{-\beta t} f(X_t^{x,y,\alpha}, \alpha_t) dt + e^{-\beta T^{x,y,\alpha}} \psi(X_{T^{x,y,\alpha}}^{x,y,\alpha}) \right),$$

$$T^{x,y,\alpha} = \inf \left\{ t \geq 0 : X_t^{x,y,\alpha} \notin G \text{ or } Y_t^{x,y,\alpha} = 0 \right\}, \quad (x,y) \in \overline{G} \times [0,\infty).$$

Here $\mathcal{U}$ is the set of all progressively measurable processes $\alpha_t \in [\underline{u}, \overline{u}]$.

Assume that ∂G is of class C^2, and let $n(x)$ be the unit outer normal to ∂G at x. Suppose that the diffusion matrix does not degenerate along the normal direction to the boundary: $\sigma^T(x,a)n(x) \neq 0$. Additionally, we assume that the functions b, σ, f are Lipschitz continuous at 0 with respect to the control variable uniformly in the state variable.

Theorem 1 *The value function v is a unique bounded viscosity solution of the Hamilton-Jacobi-Bellman (HJB) equation*

$$\beta v + \inf_{\alpha \in [\underline{u}, \overline{u}]} \left\{ -f(x,a) - b(x,a) v_x - \frac{1}{2} \mathrm{Tr}(\sigma(x,a)\sigma^T(x,a) v_{xx} + |a| v_y \right\} = 0,$$

$(x,y) \in G \times (0,\infty)$*, which is continuous on* $\overline{G} \times [0,\infty)$ *and satisfies the boundary conditions:*

$$v(0,x) = \psi(x), \quad x \in \overline{G}; \quad v(x,y) = 0, \quad x \in \partial G, \quad y \geq 0.$$

The main point is to proof that v is continuous at the boundary points $(x,0), x \in G$, since at these points the HJB equation degenerates in a specific way, which does not allow to apply the known results [1, 3] directly.

We consider a couple of examples, where the state variable X is a controlled Ornstein-Uhlenbeck process:

$$dX = k(l - X_t)dt + \sigma dW_t - \alpha_t dt,$$

and $f = 1$. The resulting functional $J = (1 - \mathsf{E} e^{-\beta T^{x,y,\alpha}})/\beta$ can be classified as a risk-sensitive criterion, related to the maximization of the expected exit time. The mentioned examples concern the optimal regulation and optimal tracking problems. We use monotone difference schemes [4] to solve these problems numerically. Theorem 1 allows to establish the convergence of the schemes by the customary Barles-Souganidis argumentation [2].

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The talk is presented on 26th of March

Numerical Probabilistic Approaches To Solution Of The Cauchy Problem For Semilinear Parabolic Equations.

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Abstract

Consider the Cauchy problem for semilinear parabolic equations

$$\frac{\partial u}{\partial t}+\frac{1}{2}A^2(x,u)\frac{\partial^2 u}{\partial x^2}+a(x,u)\frac{\partial u}{\partial x}+g(x,u)=0, \tag{1}$$

$$u(T,x)=u_0(x). \tag{2}$$

Here $x\in R^1,\ t\in[0,T]\,,\ u(t,x)\in R^1,\ A(t,x)\in R^1,\ a(t,x)\in R^1,\ g(x,u)\in R^1$.

Problems of the form (1), (2) arise as mathematical models of various phenomena in physics, biology, financial mathematics and other fields. There are exist at least two probabilistic approaches to solution of such problems (see [1; 2]). According to these approaches one can construct two probabilistic representations of the solution of the problem (1), (2). We aim to construct and implement numerical algorithms to construct classical and viscosity solutions of the above Cauchy problem.

We assume that $u_0(x)$ is a smooth bounded function, coefficients a, A and g are sublinear and satisfy the Lipschitz condition in all arguments.

Let $w(s)\in R^1$ be a standard Wiener process defined on a probability space (Ω,F,P).

It is known [1] that the counterpart of the Cauchy problem (1), (2) is given by

$$\begin{cases} d\xi(s)=a(\xi(s),u(s,\xi(s)))ds+A(\xi(s),u(s,\xi(s)))dw(s),\xi(t)=x\\ u(t,x)=E[u_0(\xi_{t,x}(T))+\displaystyle\int_t^T g(\xi(s),u(s,\xi(s)))ds],0\le t\le s\le T\end{cases} \tag{3}$$

and one can use (3) to construct a classical solution of the problem (1), (2).

There exists an alternative approach [2] leading to a system of forward SDE and backward SDE (FBSDE)

$$\begin{cases} d\xi(s)=a(\xi(s),y(s))ds+A(\xi(s),y(s))dw(s),\quad \xi(t)=x\\ dy(s)=-g(\xi(s),y(s))ds+z(s)dw(\theta),\quad y(T)=u_0(\xi(T)),0\le t\le s\le T.\end{cases} \tag{4}$$

Relation $y(t)=u(t,x)$ gives rise to a continuous viscosity solution of the Cauchy problem (1), (2). Here $y(s)=u(s,\xi(s)),\quad z(s)=A(\xi(s),\quad y(s))\frac{\partial u}{\partial x}(s,\xi(s))$ solve FBSDE (4).

We consider numerical algorithms to construct functions $u(t,x)$ given by (3) and (4), that is to construct numerical solutions of the problem (1), (2) based on the results from [3] and [4].

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The talk is presented on 25th of March

A Practical Guide To The Analysis Of Derivatives.

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Abstract

In the past years the number of lawsuits related to derivatives increased substantially. Thereby the demand for expert opinions rose to answer questions related to market value, risk and the suitability of derivatives. In this presentation we will provide some practical examples and arising challenges.

The talk is presented on 27th of March

Infinite Horizon Optimal Control Problems And Pseudospectral Methods For The Solution.

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Abstract

Still at the beginning of the previous century the optimal control problems with infinite horizon became very important. Applications in economics and biology show, that an infinite horizon seems to be a very natural phenomenon. Different optimality concepts were introduced, see e.g. [1] and corresponding optimality criteria were obtained. The usual Pontryagin type Maximum Principle (PMP) cannot be easily adjusted to the case of infinite horizon problems as it was first demonstrated in an example of Halkin [2]. In this paper we prefer to use a strong optimality criterion, see [1].Our key idea is to introduce a weighted Sobolev space $W_2^1(\mathbb{R}_+, \mu)$ as state space and a weighted Lebesgue space $L_2(\mathbb{R}_+, \mu)$ as control space. In this setting the linear-quadratic regulator problem becomes an optimization problem in Hilbert spaces and we can use classical tools of convex analysis. A Pontryagin type Maximum Principle, which includes a transversality condition, as well as existence results were established in [3; 4].

The treatment of the infinite horizon problem as an optimization problem in Hilbert spaces suggests to develop a Galerkin type method for its numerical solution, i.e. to look for a solution of canonical system as a series expansion with unknown coefficients with respect to some complete orthonormal system of elements. Combining this technique with application of the Gauss-Laguerre quadrature formula for the approximate evaluation of the functional value results in the so called indirect pseudospectral method. Its development is the main issue of this paper.

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The talk is presented on 26th of March

Model Of Optimal Flood Hydrograph Social Natural Management Of System "Volga Hydroelectric Power Station - Volga-Akhtuba Floodplain".

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Abstract

As a result of cognitive analysis of the "Volga HPP - Volga-Akhtuba floodplain" system the problem of the conventional multi-criteria optimization of flood hydrograph Volga hydroelectric power station is formulated on the set of single-stage and two-stage hydrographs. Developed and implemented a numerical algorithm for constructing the Pareto-optimal set of solutions of this problem with the use of analytical functions describing the flood hydrological regime of the Volga-Akhtuba floodplain.

The purpose of this paper is to build an optimal flood hydrograph Volga-Akhtuba floodplain on the set of single-stage and two-stage hydrograph using knowledge about the hierarchy social natural management of system "Volga HPP - Volga-Akhtuba floodplain" (SNMS "VHPP-VAF") and the results of the study of the dynamics of flood inundation Volga-Akhtuba floodplain (VAF). The object of study is the problem of progressive dehydration northern part of the Volga-Akhtuba floodplain (VAF) [1] and the possible negative impact of changes by changing the discharge regime VHPP (hydrograph).

Variables and parameters of the hydrological regime are: $S_{in}(t)(S_{min}, S_{max})$ - the current (minimum, maximum allowable) area of flood inundation, $Q(t)(Q_{max})$ - current (maximum) flow rate, V-volume of flood hydrograph, $S_{sp}(t)(S_{sp}^{min}, S_{sp}^{max}, S_{sp}^{opt})$ - real (minimum, maximum, optimum allowable) spawning area, $T_{sp}(t)(T_{sp}^{min}, T_{sp}^{max}, T_{sp}^{opt})$- real (minimum, maximum, optimum allowable) spawning period, $S_{env}^{opt}(S_{ec}^{opt}, S_{soc}^{opt})$ - environmentally (economically, socially) optimal flooded area, hydrological safety VGHPP (region).

Subject to security constraints, we can construct an optimization problem of constrained optimization flood hydrograph:

$$\begin{aligned} V \to \min_{G}, \quad S_{in} = \min(S_{env}^{opt}, S_{ec}^{opt}, S_{soc}^{opt}), \quad T_{sp} \to T_{sp}^{opt}, \quad T_{sp}^{min} \le T_{sp}^{opt} \le T_{sp}^{max}, \\ S_{sp} \to S_{sp}^{opt}, \quad S_{sp}^{min} \le S_{sp}^{opt} \le S_{sp}^{max}, \quad Q(t) \le Q_{max}, \quad S \le S_{max}, \end{aligned} \tag{1}$$

where $G = \{Q1, \tau_1, Q2, \tau_2\}$-hydrograph (Figure 1).

To solve the problem (1) use an analytical approximation of relief, hydraulic transient dynamic characteristics of flood inundation VAF [2]. Was studied numerically single and two-stage models hydrograph and a comparative analysis of the use of hydrograph for floodplain ecosystem.

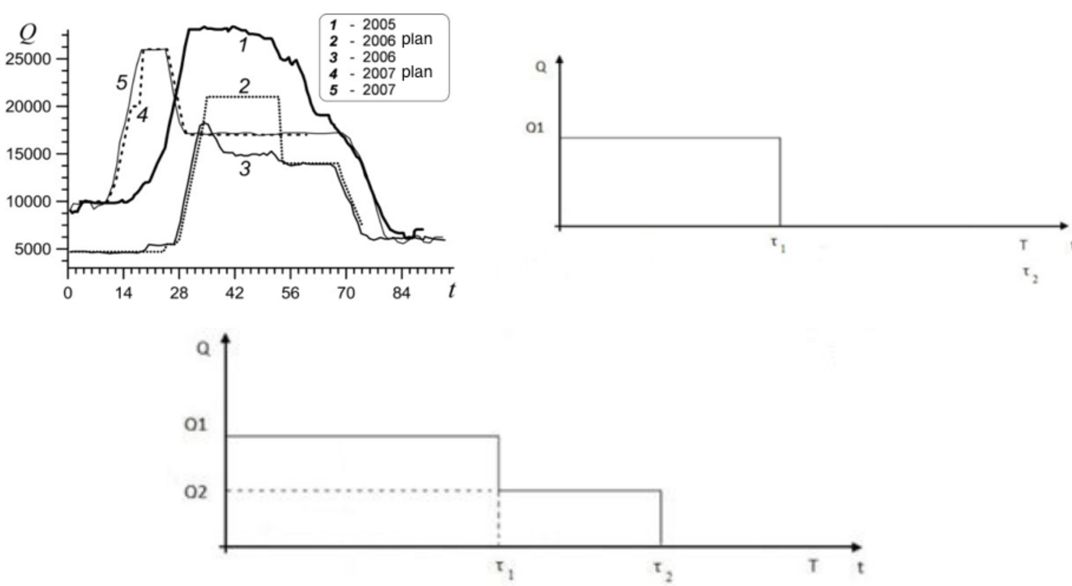

Figure 1: Actual and planned hydrographs 2005-2007. (left), the planned single-stage hydrograph (right), a planned two-stage hydrograph (bottom).

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The talk is presented on 23th of March

Delay-differential Equations And Large Delay Approximation.

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Abstract

Introduction of delayed feedback term into ODEs and PDEs often leads to additional bifurcations and novel dynamics. To study the spectrum of such systems, one needs to solve transcendental eigenvalue problem, which only in particular cases can be simplified with the use of Lambert W function. Large delay approximation may greatly aid the analysis of delay-differential equations, allowing to study their dynamics analytically or reduce the numerical complexity of the problem.

The talk is presented on 23th of March

Verification Problem For Viscosity Solutions And Stochastic Perron's Method.

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Abstract

1. It is well known that for controlled diffusion processes the value function satisfies the related second order Hamilton-Jacobi-Bellman (HJB) nonlinear partial differential equation in the viscosity sense. The traditional way to prove this fact is based on Bellman's dynamic programming principle (DPP). The first, and often difficult step, (i) is to prove the dynamic principle itself. The second step (ii) is to apply it to test functions and pass to the limit. From the resulting inequalities eventually one can conclude that the value function is a viscosity solution of the HJB equation. The third step (iii) is to establish *a comparison result*, implying that the value function is a unique continuous viscosity solution of the HJB equation.

Steps (i)-(iii) can be considered as a verification procedure in the framework of viscosity solutions. In the classical scheme one starts from the HJB equation and, under quite restrictive assumptions, identify its smooth solution with the value function.

There are several variations of the above scheme. (a) The DPP can be modified. The weak dynamic programming principle [3] is powerful enough to perform the step (ii) above. (b) In many situations it is possible to perform steps (i) and (ii) for a simpler problem with a penalty term, and then pass to the limit. (c) If a comparison result is not available, at step (iii) one can often identify the value function with a viscosity solution with some "extremal" properties [1]. (d) The classical verification procedure can be adapted to the viscosity case [9].

2. The stochastic Perron method (SPM), invented by E. Bayraktar and M. Ŝirbu [2], does not use a DPP, and avoids a direct study of the value function. It works with the families $\mathcal{V}_-, \mathcal{V}_+$ of *stochastic* sub- and supersolutions, which produce sub- and supermartingale-like processes by the superposition with the state process, and bound the value function from below and above: $u \le v \le w,\ u \in \mathcal{V}_-,\ w \in \mathcal{V}_+$. The main point of the SPM is the fact that

$$u_-(x) := \sup_{u\in\mathcal{V}^-} u(x), \qquad w_+(x) := \inf_{w\in\mathcal{V}^+} w(x)$$

are respectively *viscosity* super- and subsolutions of the HJB equation. Then, under the purely analytical assumption that a comparison result holds true, one can conclude that a unique (continuous) viscosity solution coincides with v.

The SPM was already applied to linear parabolic equations, stochastic differen- tial games, regular, singular and state constrained control problems.

3.Our main example is the exit time control problem, studied in [6]. Consider the system of controlled stochastic differential equations:

$$dX_s = b(X_s, \alpha_s)ds + \sigma(X_s, \alpha_s)dW_s, \quad X_0 = x,$$

where W is a standard m-dimensional Brownian motion with respect to some filtration $\mathbb{F} = (\mathscr{F}_s)_{s\ge 0}$, satisfying the usual conditions. The control process α takes values in a

compact subset A of $\mathbb{R}^k$ with $0 \in A$, and is assumed to be progressively measurable with respect to $\mathbb{F}$. Denote by $\mathcal{A}$ the set of such controls. We assume that the stochastic control problem is in the standard form (see [8], Chapter 3).

Let G be a connected open set in $\mathbb{R}^d$. Consider a Borel set $\widehat{G}$ between G and its closure: $G \subseteq \widehat{G} \subseteq \overline{G}$, and denote by $\tau^{x,\alpha} = \inf\left\{s \geq 0 : X_s^{x,\alpha} \notin \widehat{G}\right\}$ the exit time of $X^{x,\alpha}$ from $\widehat{G}$. The value function v of the corresponding stochastic control problem is defined as follows

$$v(x) = \sup_{\alpha \in \mathcal{A}} \mathsf{E}\left(\int_0^{\tau^{x,\alpha}} e^{-\beta s} f(X_s^{x,\alpha}, \alpha_s) ds + e^{-\beta\tau^{x,\alpha}} g(X_{\tau^{x,\alpha}}^{x,\alpha})\right).$$

Here $\beta > 0$ and the functions $f : \overline{G} \times A \mapsto \mathbb{R}, g : \partial G \mapsto \mathbb{R}$ are continuous and bounded.

From the theory of stochastic optimal control it is known that v should satisfy the Dirichlet boundary value problem

$$\beta v - \sup_{a \in A}\left[f(x,a) + b(x,a) \cdot Dv + \frac{1}{2}\mathrm{Tr}(\sigma(x,a)\sigma^T(x,a)D^2 v)\right] = 0, \quad x \in G, \tag{1}$$

$$v(x) = g(x), \quad x \in \partial G, \tag{2}$$

where the HJB equation, as well as the boundary condition, should be interpreted in the viscosity sense [4].

Assume that the strong comparison result (SCR) holds true: that is, $u \leq w$ on G for any bounded viscosity subsolution u and supersolution w of (1), (2). The following result was proved in [6] by the stochastic Perron method.

Theorem 1 *Under the SCR v is continuous on G and $v(x) = \widetilde{v}(x)$, $x \in G$ for any viscosity solution $\widetilde{v}$ of (1), (2).*

We also plan to discuss some concrete examples and the state constrained problem, where a strategy α is admissible if $X_t^{x,\alpha} \in \overline{G}, t \geq 0$. In the latter case the SPM method allows to obtain a result similar to [5] (see [7]).

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The talk is presented on 24th of March

Martingale Approach To Utility Maximization In Jump Models With Differential Rates And Marked Point Processes.

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Abstract

We explore martingale and convex duality techniques to study optimal investment strategies that maximize expected risk-averse utility from consumption and terminal wealth in a market model driven by multivariate marked point processes. We assume the wealth equation is nonlinear with respect to the portfolio proportion process, as in the case of differential rates for borrowing and lending. We provide sufficient conditions for existence of optimal policies and find closed-form solutions for the optimal value function in the case of pure-jump models with jump-size distributions modulated by a two-state Markov chain and agents with logarithmic and fractional power utility.

Paper is available on arxiv: http://arxiv.org/abs/1411.1103

The talk is presented on 26th of March

Partially Observable Stochastic Optimal Control Problems For An Energy Storage.

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Abstract

We address a real option problem consisting of the valuation of a energy storage facility in the presence of stochastic energy prices. Such problems arise in case of natural gas dome storage and hydroelectric pumped storage. The valuation problem is related to the problem of determining the optimal injection/withdrawal strategy that maximizes the expected value of the resulting discounted cash flows over the lifetime of the storage. Special features are an energy price model containing an unobservable factor process driving the mean-reversion level, operational constraints on the strategy and the storage level.

We formulate the problem as a stochastic control problem in continuous time, resulting in a Hamilton-Jacobi-Bellman (HJB) equation for the value function. We use numerical methods such as policy improvement and finite difference schemes for computing approximations of the value function and the optimal strategy. Finally, we illustrate our results with some numerical examples.

The talk is based on joint work with Ralf Wunderlich.
Keywords: stochastic optimal control, HJB equation, energy markets

The talk is presented on 26th of March

Bayesian Disorder Problems On The Filtered Probability Spaces.

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Abstract

In the talk we investigate the general bayesian disorder problems which generalized many problems in the literatures. We investigate the properties the basic statistics which show how disorder problems can be reduced to the optimal stopping problems.
Our general results give a possibility to study disorder problems for Brownian motion on FINITE time interval.

It is joint work with M. Zitlukhin.

The talk is presented on 24th of March

Pontryagins Maximum Principle For Infinite Horizon Optimal Control Problems With State Constraints.

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Abstract

Optimal control problems with infinite horizon arise in many fields, in particular in the economic growth theory. Typically, the performance index is described by an integral on an unbounded interval. In this talk we present the Pontryagins maximum principle for the infinite horizon optimal control problem with state constraints: Minimize the functional

$$J(x(\cdot), u(\cdot)) = \text{(L-)} \int_0^\infty f(t, x(t), u(t)) e^{-\varrho t} dt \to \inf$$

(herein (L-) $\int$ denotes the Lebesgue-integral) subject to the state equation

$$\dot{x}(t) = \varphi(t, x(t), u(t)), \quad x(0) = x_0,$$

the state constraints

$$g_j(t, x(t)) \leq 0 \;\; \forall t \in \mathbb{R}_+, \quad j = 1, ..., l,$$

and control restrictions

$$u(t) \in U \subseteq \mathbb{R}^m, \quad U \neq \emptyset.$$

We consider this optimal control problem for measureable and bounded controls $u(\cdot)$ and for states $x(\cdot)$ which belong to the Weighted Sobolev space $W_2^1(\mathbb{R}_+, \mathbb{R}^n; \nu)$:

$$W_2^1(\mathbb{R}_+, \mathbb{R}^n; \nu) := \left\{ x(\cdot) \,\middle|\, \int_0^\infty (\|x(t)\|^2 + \|\dot{x}(t)\|) \nu(t) dt < \infty \right\}, \quad \nu(t) = e^{-at}, a > 0.$$

For the class of problems proposed, we state the Pontryagin maximum principle and give applications. The obtained Pontryagin maximum principle includes the adjoint equation, the maximum condition and also transversality conditions.

The talk is presented on 26th of March

Cluster analysis.

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Abstract

The cluster analysis is a well-known member of the unsupervised learning methods family. However, using clusterization algorithms in practice could be tricky and nontrivial. In this talk I will make a brief overview of clusterization methods such as centroid-based clustering, hierarchical clustering, graph partition and then give several practical advices and clues with examples from the real world large-scale project made in Yandex.

The talk is presented on 27th of March

A Kinetic Equation For Modelling Irrationality And Herding Effects.

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Abstract

In financial crises, often herding behaviour, which is characterized by prize bubbles and crashes, can be observed. This behaviour may be caused by an irrational behaviour of the market participants, e.g. driven by emotion. We suggest a simple kinetic equation based on binary interactions of the market agents.

The goal is to describe the evolution of the distribution function of the value of a given product in a large market taking into account the effect of a herding behaviour (which is not possible with classical linear financial models) and the rationality of the individuals. We derive the corresponding nonlocal Fokker-Planck equation in the crazing collisions limit and we prove the existence of weak solutions. Finally, we present some numerical simulations.

The talk is presented on 25th of March

L - Stability Of Thrice Implicit Difference Schemes Of Eighth Order Of Approximation For ODEs Stiff Systems.

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Abstract

This report presents a study of the stability properties of the extended two-parameter family of thrice implicit difference schemes (3IDS) with the second derivative [1-4]. It is shown that the family has the eighth order of approximation. We also show that among many A-stable 3ISD-schemes, there are two one-parameter families: the family L-stable schemes and family of schemes of high accuracy for linear problems. Testing of these difference schemes for linear and nonlinear problems with different numbers of stiffness.

The new set of absolutely stable difference schemes for a numerical solution of ODEs stiff systems (1) is submitted.

$$\frac{d}{dt}v(t) = f(v), \quad v(0) = v_0, \quad t > 0 \tag{1}$$

The main feature of the set is the multi-implicit finite differences with the second derivatives of the desired solution. In this work studies the expanded two-parameter (α, β) set of 3ISD-schemes

$$\begin{cases} \frac{v_{n+1}-v_n}{\tau} = \sum_{i=0}^{3}(a_{1i}E + \tau b_{1i}J_{n+i})f_{n+i} \\ \frac{v_{n+2}-v_n}{\tau} = \sum_{i=0}^{3}(a_{2i}E + \tau b_{2i}J_{n+i})f_{n+i} \\ \frac{v_{n+3}-v_n}{\tau} = \sum_{i=0}^{3}(a_{3i}E + \tau b_{3i}J_{n+i})f_{n+i} \end{cases} \tag{2}$$

The coefficients for the method of providing the 8th-order approximation as whole have the form

$$(a_{ik}) = \begin{pmatrix} \frac{6893}{18114} + \frac{11}{3}\alpha & \frac{313}{672} + 9\alpha & \frac{89}{672} - 9\alpha & \frac{397}{18114} - \frac{11}{3}\alpha \\ \frac{223}{1134} + \frac{11}{3}\beta & \frac{10}{21} + 9\beta & \frac{13}{21} - 9\beta & \frac{10}{567} - \frac{11}{3}\beta \\ \frac{31}{224} & \frac{81}{224} & \frac{81}{224} & \frac{31}{224} \end{pmatrix},$$

$$(b_{ik}) = \begin{pmatrix} \frac{1283}{30240} + \alpha & \frac{-851}{3360} + 9\alpha & \frac{-269}{3360} + 9\alpha & \frac{-163}{30240} + \alpha \\ \frac{43}{1890} + \beta & \frac{-8}{105} + 9\beta & \frac{-19}{210} + 9\beta & \frac{-4}{945} + \beta \\ \frac{19}{1120} & \frac{-27}{1120} & \frac{27}{1120} & \frac{-19}{1120} \end{pmatrix}$$

At arbitrary (α, β) parameters last difference equation in (2) has 8-th order of approximating.

We found that family of absolutely stable 3ISD-schemes includes two one-parameter families: the set of the L-stable schemes and the set of the schemes of heightened accuracy for linear problems. For example, there are

- the A-stable scheme of 10-th order of approximation, if $\alpha = \frac{1}{540},\ \beta = \frac{1}{1080}$,
- L_1 - stable scheme with 9-th order of approximation, if $\alpha = \frac{1}{54},\ \beta = -\frac{1}{135}$,
- L_2 - stable scheme with 8-th order of approximation, if $\alpha = \frac{1}{54},\ \beta = -\frac{1}{126}$.

These difference schemes have tested on linear and non-linear problems with different stiffness numbers. In numerical experiments are computed the errors of a numerical solution as functions of integration step. These results demonstrate high quality of stability and accuracy of the suggested 3ISD-schemes.

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The talk is presented on 25th of March

Valuation Of Asia Options By Implicit Difference Method.

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Abstract

Popularity options as derivative financial instruments, increases and stimulates the development of mathematical methods for their valuation. Currently, the stock market has numerous types of options: European, American, barrier, Exotic, etc. This report focuses on evaluating the Asian option. Mathematical model of the problem is the Black-Scholes model, which is a parabolic partial differential equation relative to the price Asian option [1].

The use of implicit finite difference schemes for solving the problem can produce a stable numerical solution for different values of volatility, risk-free rate and the time of exercise [1; 2]. For numerical solution algorithm was developed, implemented as a program Asia option programming language C++. These calculations for the analysis of the results generated in the form of tables and graphs in spreadsheet format and Excel charts.

The program Asia option is optional calculator, designed to calculate the value of Asian option based on the numerical solution of the Black-Scholes. Risk-free interest rate, volatility, initial price of the asset and expiration are input data for numerical calculations.

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The talk is presented on 23th of March

Risk Modelling In Equities: Theory Vs. Practice

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Abstract

Working in the financial industry for a few years now, I have noticed that often there is a gap between the way the fresh graduates think of what they will be doing as quantitative analysts in the industry and what is expected from in reality. In this talk we will try to bridge that gap, focussing on the low-frequency multi-factor "Fama-French style" models. Such models are widely used in the investment banks and buy-side firms to analyze the systematic risk in the equity (stock) portfolios and drivers behind their performance. We are going to go through the basics of such models, have a quick look at the typical factors affecting the stock risk and performance, go through some of the useful applications and finally discuss some of the typical problems that arise when one tries to build such model.

The talk might be more interesting to the students and fresh graduates with limited experience in the industry that are considering becoming quants in the financial institutions.

The talk is presented on 27th of March

Compact Difference Schemes for Option Pricing Liquidity Shocks Models.

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Abstract

This paper provides a numerical investigation for European options under parabolic-ordinary system modeling markets to liquidity shocks. Our main results concern construction and analysis of fourth-order and sixth-order compact finite difference schemes. We present numerical results to illustrate the high efficiency of the compact difference schemes.

Keywords: Option pricing, finite difference scheme, high-order compact difference scheme, discrete comparison principle, convergence.

The talk was presented on 25th of March

Structural Models Of Spot Electricity Markets.

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Abstract

The structural approach to electricity markets investigates the intricacies of the wholesale electricity price formation by quantifying the impact of employed generation technologies and their production costs, fluctuations of the demand and production capacities, the influence of renewable energy sources, weather, and so on. We develop a general structural framework of supply and demand, recast some of the existing models of electricity spot markets into this framework, and sketch several interesting mathematical problems arising naturally in this context.

The talk is presented on 24th of March

Portfolio Optimization With An Exogenous Random Liquidation Time.

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Abstract

It is extremely important to take a low liquidity of an asset into consideration when operating on a financial market. The buyer proposes the price that can differ greatly from the paper value estimated by the seller, so the seller cannot liquidate his portfolio instantly and waits for a more favorable offer. To minimize losses and move the theory towards practical needs one can take into account the time lag of the liquidation of an illiquid asset. Working in the Merton's optimal consumption framework with continuous time we consider an optimization problem for a portfolio with an illiquid, a risky and a risk-free asset. While a standard Black-Scholes market describes the liquid part of the investment the illiquid asset is sold at an exogenous random moment with prescribed liquidation time distribution. The investor has the logarithmic utility function as a limit case of a HARA-type utility. Different distributions of the liquidation time of the illiquid asset are under consideration - a classical exponential distribution and Weibull distribution that is more practically relevant. We show the existence of the viscosity solution in both cases. Applying numerical methods we compare classical Merton's strategies and the optimal consumption-allocation strategies for portfolios with different liquidation time distributions of an illiquid asset. We demonstrate how optimal strategies depend on the values of the parameters of the model and explain the economical meaning of the demonstrated dependences.

The talk is presented on 26th of March

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