

Preface

This text book is devoted to the Lie group analysis of differential equations with applications to Financial Mathematics. The new models arising in Financial Mathematics are often presented in the form of nonlinear partial differential equations (PDEs). The Lie group analysis was earlier successfully applied to the studies of algebraic and group structures of linear and nonlinear PDEs which describe physical or chemical processes. It is to expect that these methods will be also used in application to new types of PDEs appearing in Financial Mathematics.

The content of this book was developed on the basis of the lecture courses *Geometrical properties of differential equations* given in the frame work of the master program "Master in Financial Mathematics" at Halmstad University, Sweden from the academic year 2006/2007 till 2010/2011. The one year program was oriented on mathematicians, physicists and computer sciences engineers who wanted to learn actual methods and modern models in Financial Mathematics.

The goal of this and others lecture courses in the master program was to provide a set of practical skills which allowed the students to use the studied methods in industry. So the main focus of the program was on a clearly understanding of the main ideas and self confident use of learned tools instead on the deep theoretical proofs and theorems. We had about 20-30 students per year. The program was very international and the students were coming from Armenia, China, Germany, Lithuania, Pakistan, Poland, Russia, Serbia, Singapore, Slovakia, Sweden, Tunis, Turkey. The atmosphere in the master program was very friendly, student supported each other. As far as I know many students stay in touch years after finishing their studies working in finance institution in different countries all over the world. Despite different skills and educational background the students in my lecture course were really enthusiastic to learn new ideas and apply them in practice.

At Halmstad university the complete academic year was divided in four periods with eight teaching weeks each and one or two examination weeks in between. During the first period I provided every week four academic

hours of lectures and four academic hours of classroom tutorials, students had assignments every week each assignment having five tasks to solve. The tasks and problems included in the assignments, class room tutorials and exams are included in this book accompanied with detailed solutions. In the last, fourth, teaching period of the study year all students wrote a master project (15 ECTS points) devoted to one of the actual problems in Financial Mathematics. These projects are published on the home page of the library of Halmstad University. Partly the results of the master projects were later published as journal papers. Some students used the Lie group analysis and studied new financial models with this method.

Later in Germany I used these lecture notes during the *Compact course on Lie group analysis* that took place in Zittau, 2013, in the framework of FP 7 Marie Curie Initial Training Network (ITN) STRIKE. The aim of the project ITN STRIKE is to understand complex (mostly nonlinear) financial models and to develop effective and robust numerical schemes for solving linear and nonlinear problems arising from the mathematical theory of pricing financial derivatives and related financial products. This two weeks intensive course was given for PhD students of different European countries working in the ITN STRIKE. The lectures were presented by Ljudmila A. Bordag, the practical exercises and assignments were lead by Ivan P. Yamshchikov. Because the participants were interested in applications of the new knowledge to numerical schemes we emphasized this part. We discussed how the admitted Lie algebraic structure of studied equations can be used to provide improved numerical schemes, how to get and use the invariant solutions and to get in touch with new models. In two weeks the participants got a tool which they can practically use.

In my opinion this book with this large amount of solved problems can be used as a text book for a regular lecture course, or for a compact lecture course during few weeks or as a book for a convenient self-study.

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Ljudmila A. Bordag
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