

Abstract

This textbook is a short comprehensive and intuitive introduction to the Lie group analysis of ordinary and partial differential equations. This practical oriented material contains a large amount of examples and problems accompanied by detailed solutions and figures. In comparison with the known beginner guides to Lie group analysis the book is oriented on students who are interested in mathematical finance and economics. It means that any physical intuition to describe the main ideas of Lie group analysis is avoided for convenience of the students. Following ideas are developed step-by-step: point transformation, meaning of a continuous one parameter group, infinitesimal action, invariants, Lie algebra, symmetry reductions of differential equations. In the book we look on differential equations from geometrical point of view, explain the idea of invariant solutions and symmetries. The admitted symmetries will be used to find the reductions of given differential equations and invariant solutions accompanied by a large amount of examples.

The book contains nine chapters. First several chapters are devoted to development of the ideas and tools of Lie group analysis. All notations, ideas and methods are explained first with very simple examples of ordinary differential equations and only then with partial differential equations. This structure of the course allows the students to get familiar with the main tools of the Lie group analysis as quickly and as easy as possible.

In Chapter 8 we study the famous Black-Scholes model for option pricing. Its algebraic structure is compared with the structure of the heat equation. The explicit analytic solution for a call option is derived simply as an invariant solution of Black-Scholes equation. In Chapter 9 we provide the results of the Lie group analysis of actual models in Financial Mathematics using the recent publications. These models are usually formulated as nonlinear partial differential equations and are rather difficult for investigations. With the help of Lie group analysis it is possible to describe some important properties of these model and get some interesting reductions in an clear and understandable algorithmic way.

The material of the book was used during 5 years within the master pro-

gram Master in Financial Mathematics at Halmstad University, Sweden and in two weeks intensive compact course given in the framework of FP 7 Marie Curie Initial Training Network (ITN) STRIKE. The participants of the compact course were PhD students primarily interested in new effective numeric schemes for advanced models in financial mathematics. As prerequisites for this text book one needs to know some basics of the theory of differential equations which corresponds usually to the bachelor level in mathematics or applied mathematics. The book can be a short introductory for a further study of modern geometrical analysis applied to models in financial mathematics. The book can be used as textbook in a master program, in an intensive compact course or for a self-study.